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Proving the Law of Conservation of Energy Using the Properties of Parallelogram

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Abstract— In physics, Law of Conservation of Energy states that energy can only be transformed from one form to another, it can neither be created nor destroyed and as such the total energy in this Universe remains constant. We see various kinds of energy transformations in our day to day lives. In all such cases energy in one form transforms into another form of energy, but in no case energy is created or destroyed. This law is true for all situations and for all kinds of transformations. There is no general proof of this law but it has been verified in many cases and as such is accepted as true. This law was first stated by Robert Mayor, a German Physicist in 1842 and was firmly established by Helmholtz. My paper is about the simple and new method for Proving the Law of Conservation of Energy using the properties of Parallelogram. This new method can be used to calculate the value of potential energy or the kinetic energy at any instant or at any given height easily. We can also find the difference between the Potential Energy and the Kinetic Energy, using this new method. I am sure that this method serves the science community in a better way.

Index Terms— Acceleration, Energy Transformation, Kinetic Energy, Law of Conservation of Energy, Mechanical Energy, Potential Energy, Properties of Parallelogram.

1 Introduction

In physics, Law of Conservation of Energy states that Energy can only be transformed from one form to another, it can neither be created nor destroyed and as such the total energy in this Universe remains constant. We see various kinds of energy transformations in our day to day lives. In all such cases energy in one form transforms into another form of energy, but in no case energy is created or destroyed. There is no general proof of this law but it has been verified in many cases and as such is accepted as true.

2 Law of Conservation of Mechanical Energy in case of a freely falling body

Let us consider an object of mass m placed at a height h from the ground. Let us neglect the effect of air resistance on the motion of the body.

Here $h = AB$ = Height of the body above the ground
s = distance of any point C from A
g = acceleration due to gravity at the place
v_1 = velocity of the body at C
v = velocity of the body at B, a point just above the ground

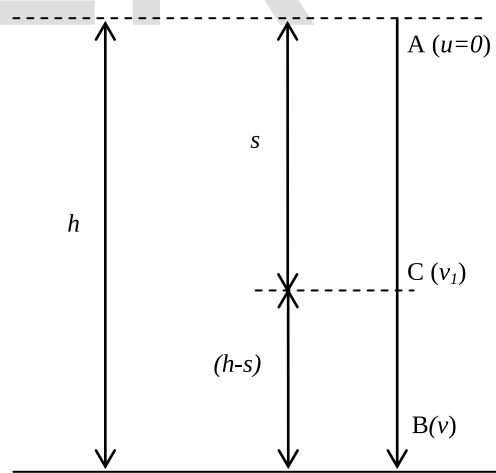
The velocity at point A is *zero, i.e.,* $u = 0$

(i) At the point A :

Potential Energy (P.E.) = mgh

Kinetic Energy (K.E.) = 0

Total Mechanical Energy (E_a) = (P.E.) + (K.E.)
$= mgh + 0$
$= mgh$ **(1)**



(ii) At the point C :

When the body moves from A to C, it covers a distance s. If v_1 is the velocity at C, then from,

$v^2 - u^2 = 2as$, we get,

$v_1^2 – 0 = 2gs$ $[v = v_1, u=0, a=g]$

Kinetic Energy (K.E.) $= ½\, mv_1^2$
$= ½\, m(2gs)$

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$= mgs$

Potential Energy (P.E.) = mg (h-s)

Total Mechanical Energy,

(E_c) = (P.E.) + (K.E.)

$= mg\ (h\text{-}s) + mgs$

$= mgh - mgs + mgs$

$= mgh$ **(2)**

(iii) At the point *B* :

From, $v^2 - u^2 = 2as$, we get,

$v^2 - 0 = 2gh$ [$v = v,\ u=0,\ a=g,\ s=h$]

$v^2 = 2gh$

Kinetic Energy (K.E.) $= ½\ mv^2$

$= ½\ m(2gh) = mgh$

Potential Energy (P.E.) = 0

Total Mechanical Energy,

(E_b) = (P.E.) + (K.E.)

= 0 + mgh

= mgh **(3)**

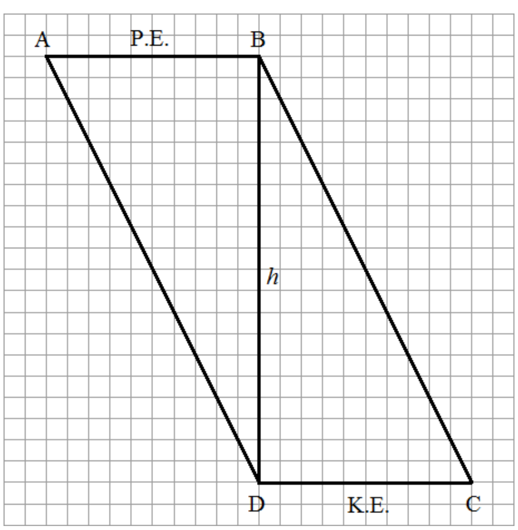
From Equations 1, 2 and 3, we can conclude that,

$E_a = E_c = E_b$

Thus the total mechanical energy of the body at the points *A*, *B* and *C* is same. The total mechanical energy is same at any other point on *AB.* As the object moves down, the potential energy goes on decreasing, whereas the kinetic energy goes on increasing. Thus, *the total mechanical energy of the body throughout the free fall is conserved.*

3 Proving Law of Conservation of Energy using the properties of Parallelogram

Let us consider an object of mass m placed at a height h from the ground. Let us neglect the effect of air resistance on the motion of the body.



Here the height of the object from ground *h*, is represented by *BD*. As there is no external force acting on the object the trajectory of the object will be *BD*. Let the flat ground be the line formed by *DC*.

Initially the object is at rest so the initial velocity *u*, will be equal to zero. We know that the Potential Energy of any object will be the maximum at maximum height. Here the Potential Energy of the body is the maximum at the point *B*.

Let the magnitude of the Potential Energy of the object be *x* at the point *B*. *AB* of *x* units is drawn perpendicular to the height. We know that, at the point *D*, the height is equal to zero. So the Potential Energy (*mgh*) of the object equals zero. Let us join *A* and *D* to get *AD.* From the figure so obtained we can notice that the Potential Energy of the object decreases with decrease in height.

The Kinetic Energy of the body at *B* is equal to zero as the object was initially at rest. At point *D* the Kinetic Energy of the body is $½\ mv^2$

Kinetic Energy of the body at the point *D* = $½\ mv^2$

Here v^2 represents the square of the velocity with which the object touches the ground. The value of v^2 can be found using the third equation of motion.

$v^2=u^2+2as$

$v^2=2as$ [$u=0$ as the object is at rest at the point B]

$v^2=2gh$ [Here the acceleration is the acceleration due to gravity; the displacement is the height from the ground]

On substituting the value of v^2 from the above equation in $½\ mv^2$ we get,

K.E. = ½ mv^2

K.E. = ½ m x $2gh$

K.E. = ½ x 2 x mgh

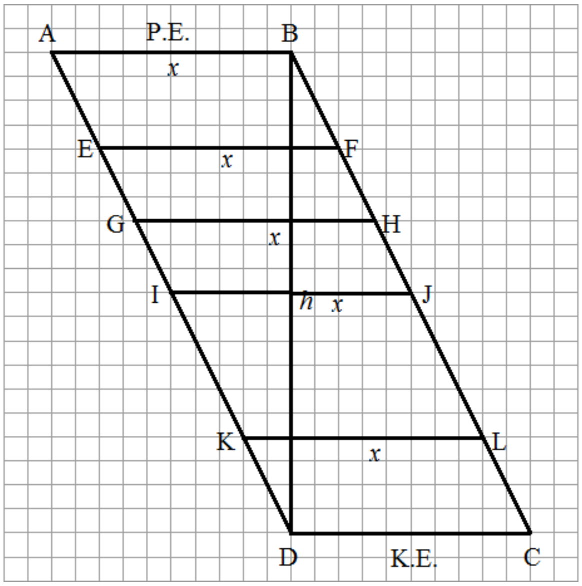
K.E. = P.E. **[P.E. = mgh]**

From this, we can say that K.E. = *x* units as P.E. = *x* units

Using this, we can draw *DC* perpendicular to *BD*, which is equal to *AB*. Let us join *B* and *C* to get *BC.*

∠ABD=∠CDB [Each 90°]

These are also the alternative interior angles formed by the transversal *BD*. As these angles are equal, *AB ∥ CD.* If opposite sides of a quadrilateral are equal and parallel then the quadrilateral is a parallelogram. So quadrilateral *ABCD* is a parallelogram. We know that the opposite sides of parallelogram are equal and parallel. Here, in parallelogram *ABCD*, the opposite sides *AD* and *BC* are parallel and equal. So the distance between the sides AD and BC along the height BD is same.



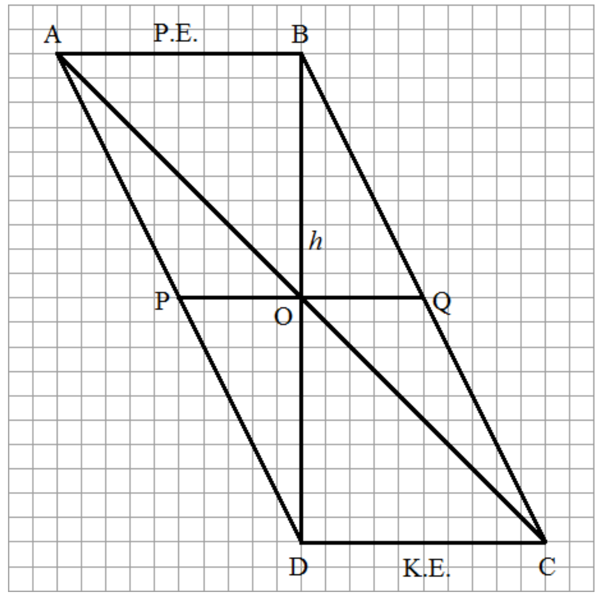
In the above figure, the distance between AD and BC is constant since these lines are parallel to each other. The measure of AB, EF, GH, IJ, KL, and DC is equal as these are perpendicular to the height and parallel lines to AB and DC.

Hence,
$AB = EF = GH = IJ = KL = DC = x$ units
We have already discussed that,
P.E = K.E. = x units
If so then,
$AB = EF = GH = IJ = KL = DC =$ **P.E.=K.E.**

But we know that the Potential Energy and the Kinetic Energy cannot remain constant throughout the journey. So our assumption on x units is wrong. It represents the sum of the Potential Energy and the Kinetic Energy of the object, or it represents the total mechanical energy of the body throughout the journey.

As we saw already that, AB = EF = GH = IJ = KL = DC = x units. Thus the total mechanical energy of the body at the points P, Q, R, and S is same. The total mechanical energy is same at any other point on BD. As the object moves down, the potential energy goes on decreasing, whereas the kinetic energy goes on increasing. This can be seen clearly when we see the distance of any point on AD from the trajectory for the Potential Energy and the distance of any point on BC from the trajectory for the Kinetic Energy. The more the distance from the trajectory, the more is the Potential Energy or Kinetic Energy. But the sum of the Potential Energy and the Kinetic Energy, i.e., the total mechanical energy which is of x units is constant. Thus, the total mechanical energy of the body throughout the free fall is conserved.

Let us now join A and C to get the diagonal AC of the parallelogram ABCD. Let it intersect the diagonal BD at O. We know that the diagonals of parallelogram bisect each other. So, here OB = OD. In fact BD is the trajectory of the body as well as the height of the body placed from the ground. So point O is half the way of the path of the object and it is equal to half the height. Let us draw another line PQ passing through O and which is parallel to AB as well as CD. As we discussed earlier, PQ represents the total mechanical energy of the body at that point.



In ΔAOP and ΔCOQ,
∠AOP = ∠COQ [Vertically Opposite Angles are equal in measure]
AO = OC [Diagonals bisect each other in a parallelogram]
∠OAP = ∠OCQ [Alternate Interior angles are equal in parallel lines]

So, ΔAOP ≅ ΔCOQ by ASA Congruency Criterion
By CPCT (Corresponding Parts of Congruent Triangles),
OP = OQ
Points P and Q are the points which represent Potential Energy and Kinetic Energy respectively. PQ represents the total mechanical energy at the point O. So we can finalize that, *the Potential Energy and the Kinetic Energy are equal when the object is half the height at which the object was present at first.*
Potential Energy at the point O = $mgh/2$= (P.E.)/2
Kinetic Energy at point O = $\frac{1}{2} mv^2$

The value of v^2 can be found using the third equation of motion.

$$v^2=u^2+2as$$

$$v^2=2as \quad [u=0 \text{ as the object is at rest at the point B}]$$

$v^2=2gh/2$ [Here the acceleration is the acceleration due to gravity; the displacement is the height from the ground]

$v^2=gh$

On substituting the value of v^2 from the above equation in ½ mv^2 we get,

K.E. = ½ mv^2

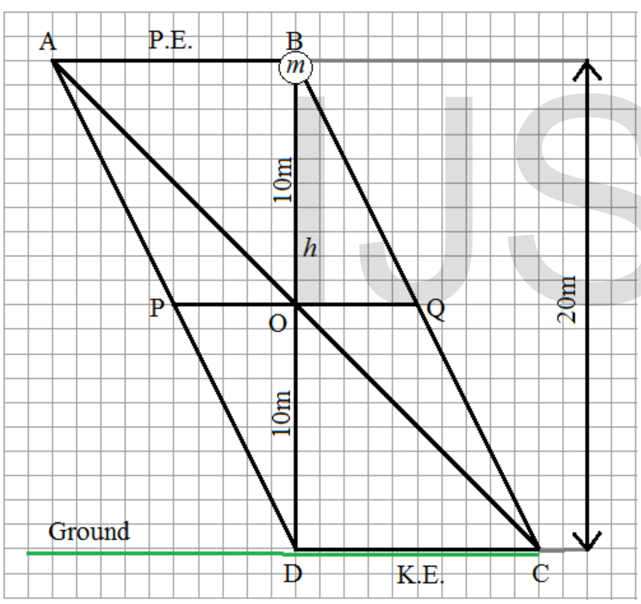
= ½ mgh

= ½ P.E.

We can find that Potential energy and the kinetic energy are equal. Hence, *the Potential Energy and the Kinetic Energy are equal when the object is half the height at which the object was present at first.*

4 Verification of this new method

Let us now verify this new method for the Law of Conservation of Energy. Let us consider an object of mass *m* placed at a height *h* from the ground. Let us neglect the effect of air resistance on the motion of the body.



Here,

Mass m = 1kg

Height h = 20m

Acceleration due to gravity g = 10ms-2

Using the above data, and the method we discussed previously, let us plot these details on a graph paper.

We need to find the Potential Energy, Kinetic Energy, and the Total mechanical energy of the body of mass m, placed at height h, at the points B, O, and D. Using the values we need to verify the Law of Conservation of energy using the new method.

(i) At the point *B*

Height h_1 = 20m

Initial Velocity u = $0ms^{-1}$

Potential Energy P.E. = mgh

= 1 x $10ms^{-2}$ x 20m

= 200 J

Kinetic Energy K.E. = ½ mv^2

= ½ x 1 x 0

= 0 J

Total Mechanical Energy,

= P.E. + K.E.

= 200 J + 0J

= 200 J

(ii) At the point *O*

Height h_2 = 10m

Velocity : $v^2=u^2+2as$

$v^2=2as$ [$u=0$ as the object is at rest at the point B]

$v^2=2gh/2$ *[Here the acceleration is the acceleration due to gravity; the displacementi is the height from the ground]*

$v^2=gh_1$

Potential Energy P.E. = mgh_2

= 1 x $10ms^{-2}$ x 10m

= 100 J

Kinetic Energy K.E. = ½ mv^2

= ½ m x gh_1

= ½ mgh_1

= ½ x 1 x $10ms^{-2}$ x 20m

= 100 J

Total Mechanical Energy,

= P.E. + K.E.

= 100 J + 100J

= 200 J

As we have discussed earlier we have verified that, *the Potential Energy and the Kinetic Energy are equal when the object is half the height at which the object was present at first.*

(iii) At the point *D*

Height h_3 = 0

Velocity : v^2 $=u^2+2as$

$v^2=2as$ [$u=0$ as the object is at rest at the point B]

$v^2=2gh_1$ [Here the acceleration is the acceleration due to gravity; the displacement is the height from the ground]

Potential Energy P.E. $= mgh_3$

$= 1 \text{ x } 10ms^{-2} \text{ x } 0m$

$= 0 J$

Kinetic Energy K.E. $= \frac{1}{2} mv^2$

$= \frac{1}{2} m \text{ x } 2gh_1$

$= mgh_1$

$= 1 \text{ x } 10ms^{-2} \text{ x } 20m$

$= 200 J$

Total Mechanical Energy.

= P.E. + K.E.

= 0 J + 200J

= 200 J

We have verified that initially the potential energy was at the maximum, which reduced gradually and on the other hand the kinetic energy which was at zero at first increased gradually as the height decreased. We have also seen that the sums of the mechanical energies are also same throughout the journey. Finally when the object reaches the ground we can see that the kinetic energy reaches it maximum while the potential energy turns to zero. Thus, *the total mechanical energy of the body throughout the free fall is conserved.*

5 Conclusion

In our day to day lives, we see many conversions of energy. In all these energy conversions, the energy is conserved according to the law of conservation of energy. The new method for proving this law using the properties of parallelograms helps us to make the calculations easily. This new method involves only plotting the given values at the first on a graph paper. This new method can be used to calculate the value of potential energy or the kinetic energy at any instant or at any given height easily. We can also find the difference between the Potential Energy and the Kinetic Energy, using this new method. We have also seen that how the point of intersection of the diagonals shows out the difference between the mechanical energies is zero. I am sure that this method serves the science community in a better way.

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